A New TLM Node for Bérenger's Perfectly Matched Layer

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Abstract—**A new two-dimensional (2-D) transmission-line matrix (TLM) node for the modeling of perfectly matched layer (PML) media is presented. A rigorous field formulation of the 2-D TLM node allows one to construct a node that has a scattering matrix fully compatible with the standard 2-D hybrid node. This approach avoids the coupling of the TLM algorithm with a finite difference time-domain (FDTD) approximation of PML field differential equations. The simulation of a wideband matched load for a WR-28 rectangular waveguide is presented for validation. A return loss better than 60 dB is obtained over the 25–40 GHz-frequency band.**

I. Introduction

The PML (perfectly matched layer) principle is based on the insertion of an absorbing layer surrounding the computational domain [1]. The PML medium is perfectly matched so that electromagnetic waves can penetrate into it at any frequency and angle of incidence, without reflection. The implementation of the PML in a numerical field modeling can be achieved by a finite-difference approximation of the PML field equations, which are then coupled with the finite-difference time-domain (FDTD) standard Yee's cell [1], [2]. For the two-dimensional (2-D) transmission-line matrix (TLM) case, an approach using the equivalence between TLM and FDTD [3], [4] to interface the TLM network and the FDTD mesh in the PML layer was presentented [5]. In this letter, the PML is considered as an active medium in which source densities depend on field components. Then, using a rigorous field formulation [6], field equations are written in terms of incident and reflected local waves to keep with the standard TLM algorithm.

II. Theory

Without loss of generality, one considers the 2-D transverse magnetic (TM) case for which only the field components E_z, H_x, and H_y are present. In a PML medium [1], E_z is decomposed into two terms, E_{zx} and E_{zy}, which transform Maxwell's curl's equations as follows:

$$\mu_d \frac{\partial H_x}{\partial t} + \sigma_y^* H_x = -\frac{\partial (E_{zx}+E_{zy})}{\partial y} \tag{1a}$$

$$\mu_d \frac{\partial H_y}{\partial t} + \sigma_x^* H_y = \frac{\partial (E_{zx}+E_{zy})}{\partial x} \tag{1b}$$

$$\varepsilon_d \frac{\partial E_{zx}}{\partial t} + \sigma_x E_{zx} = \frac{\partial H_y}{\partial x} \tag{1c}$$

$$\varepsilon_d \frac{\partial E_{zy}}{\partial t} + \sigma_y E_{zy} = -\frac{\partial H_x}{\partial y} \tag{1d}$$

where $\varepsilon_d, \mu_d, \sigma$, and σ^* are the permittivity, permeability, electric, and magnetic conductivities, respectively, of the PML layer. Setting $E_z = E_{zx} + E_{zy}$, (1c) for instance, can be rewritten as

$$\varepsilon_d \frac{\partial E_z}{\partial t} + \sigma_x E_z + (\sigma_y - \sigma_x) E_{zy} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}. \tag{1c$'$}$$

Hence, (1a), (1b), and (1c$'$) can be interpreted as describing an anisotropic medium with both electric and magnetic losses. The term $(\sigma_y - \sigma_x)E_{zy}$ can be related to an electric current source density controlled by field time and spatial derivatives in (1d). Consequently, the PML medium can be treated by the general hybrid node formulation presented in [6].

First, consider the interface perpendicular to the x-axis that separates the computational domain from the PML medium (Fig. 1), both having the same basic constitutive parameters (ε_d, μ_d). Since the attenuation along the x direction is most important, one can choose (ε_d, μ_d). Finally, in the PML layer σ_x and σ_x^* should satisfy

$$\frac{\sigma_x}{\varepsilon_d} = \frac{\sigma_x^*}{\mu_d}. \tag{2}$$

With the above condition the PML medium is perfectly matched for normal incidence. For arbitrary incidence, the source term $(-\sigma_x E_{zy})$ contributes to a correction so that no refraction and reflection occur for all frequencies.

The general field formulation of a 2-D TLM hybrid node [6] allows the modeling of electric, magnetic losses, and current densities. The source term is approximated as follows (see Fig. 2):

$$\iint_S (-\sigma_x E_{zy})\, dx \cdot dy \cong -\sigma_x \frac{\Delta_x \cdot \Delta_y}{\Delta l} \Delta l \cdot E_{zy}^{(k)} \tag{3}$$

where $E_{zy}^{(k)}$ is the value of the subcomponent E_{zy} at the cell center and sampled at time $k\Delta t, \Delta_x$, and Δ_y are the mesh size in direction x and y, respectively, and Δl takes arbitrary value and is involved in the hybrid node algorithm [6]. The term $E_{zy}^{(k)}$ can be evaluated by an approximated expression of (1d) in which $\sigma_y = 0$

$$\frac{\varepsilon_d}{\Delta l} \frac{(1-T)}{\Delta t} \cdot \Delta l \left(\frac{E_1^{zy} + E_3^{zy}}{2} \right) = \frac{1}{\Delta_x} \cdot \frac{(1+T)}{2} \cdot \Delta_x \left(\frac{H_1^x - H_3^x}{\Delta_y} \right) \tag{4}$$

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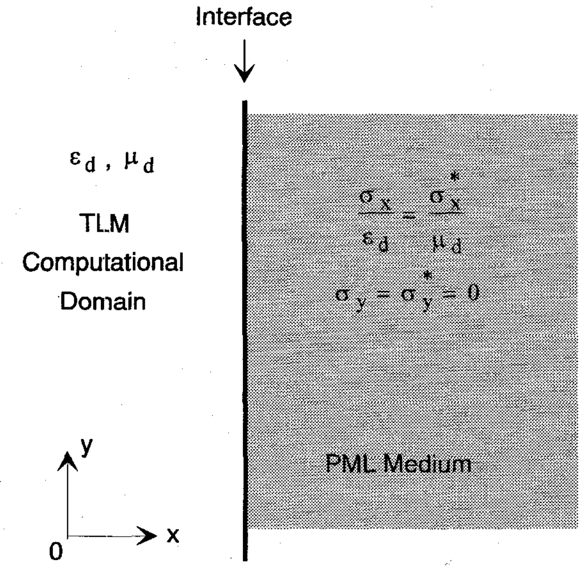

Fig. 1. Interface between the computational domain and the PML medium.

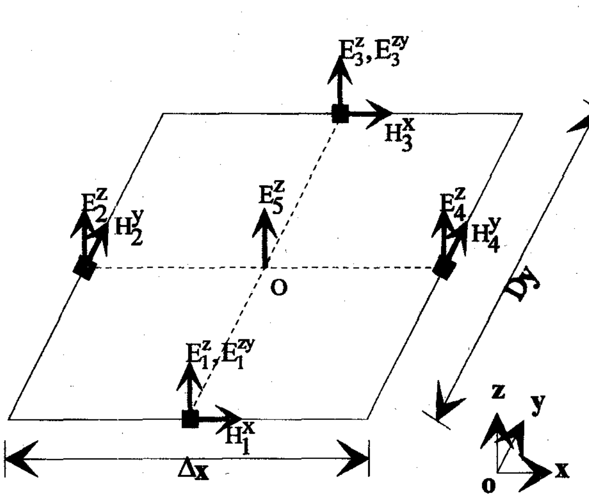

Fig. 2. Generic cell for the 2-D TLM (TM mode) for interface PML–TLM normal to the x axis.

where T is a delay operator defined by $T \cdot u^{(k+1/2)} = u^{(k-1/2)}$. Note that electric field time-derivative is applied to a spatial average value, whereas magnetic field spatial-derivative is applied to a time average value. Effects due to active sources can be inserted in a compact matrix notation by introducing two supplementary arms whose voltage components are defined according to their usual expression in TLM

$$a_6; b_6 = \frac{1}{2}(\Delta l \cdot E_1^{zy} \pm Z_{zy}\Delta_x \cdot H_1^x) \tag{5a}$$

$$a_7, b_7 = \frac{1}{2}(\Delta l \cdot E_3^{zy} \mp Z_{zy}\Delta_x \cdot H_3^x) \tag{5b}$$

where $Z_{zy} = \Delta t \cdot \Delta l/(\varepsilon_d \Delta_x \cdot \Delta_y)$. By performing on (1d) a time integration from $(k - 1/2)\Delta t$ to $k\Delta t$ and a spatial integration along the y direction from the edge to center of the cell (see Fig. 2), and then using (4) and (5), the effect of the controlled source $(-\sigma_x E_{zy})$ can be expressed in terms of voltages in arm 6 and 7. As a result, the new TLM node of a PML medium is described by a 7×7 scattering matrix. The above procedure is general and applies straightforwardly for interfaces normal to the y direction. In the situation where $\varepsilon_d = \varepsilon_0, \mu_d = \mu_0$, square mesh $(\Delta_x = \Delta_y = \Delta)$ and $\Delta t = \Delta/(c_0\sqrt{2})$ are used, the PML–TLM node is described by the following scattering matrix:

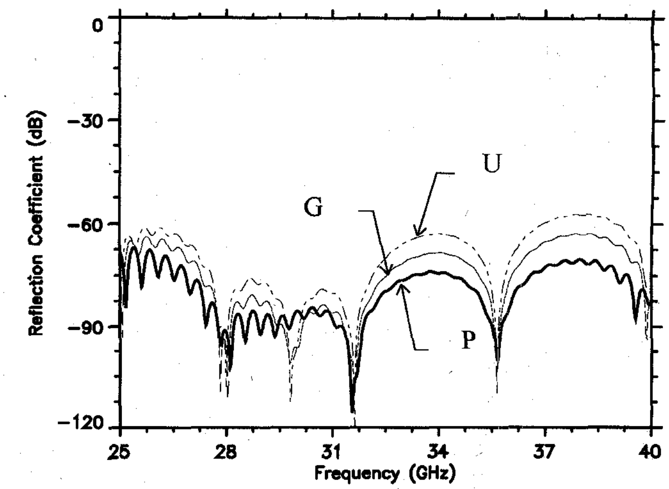

Fig. 3. Reflection coefficient versus frequency with different electrical conductivity profiles in the PML medium: Uniform (U), Geometric (G), and Parabolic (P).

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ \cdots \\ b_6 \\ b_7 \end{bmatrix}^{(k+1/2)} = \begin{bmatrix} -b & 2a & 2a & 2a & \vdots & d & d \\ 2a & -2a & 2a & -c & \vdots & d & d \\ 2a & 2a & -b & 2a & \vdots & d & d \\ 2a & -c & 2a & -2a & \vdots & d & d \\ \cdots & \cdots & \cdots & \cdots & \cdot & \cdots & \cdots \\ -1/2 & 0 & 1/2 & 0 & \vdots & 1 & 0 \\ 1/2 & 0 & 1/2 & 0 & \vdots & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ \cdots \\ a_6 \\ a_7 \end{bmatrix}^{(k-1/2)} \tag{6}$$

where $a = 1/(G_e + 4), b = (G_e + 2)/(G_e + 4), c = (G_e - 2)/(G_e + 4), d = G_e/(G_e + 4)$ and $G_e = \sqrt{2}Z_0\sigma_x\Delta$. The top left 4×4 submatrix pertains to the one of a medium with anisotropic conductivities, (ε_0, μ_0) and square mesh for the TLM. Any other situation would add the fifth arm due to hybrid node stub-loading [6]. Finally, the formulation presented here is not restricted to the TM case as duality applies to the TE case.

III. Results

The new PML–TLM node described by (12) was used for simulating a wideband matched load of a WR-28 rectangular waveguide ($a = 31\Delta = 7.112$ mm), 250Δ long, terminated at each end by a PML medium of 25Δ thickness, and backed by perfectly conducting walls. The structure was excited with a sheet of electric current located at the middle of the structure with sinusoidal spatial distribution across the section and

Gaussian pulse time variation. As a result, only the TE_{10} mode was propagating. The electric field was recorded at one point adjacent to the PML-guide interface. The reflection coefficient (a long waveguide case was used as reference) is presented in Fig. 3 with different electric conductivity profiles of the PML. They are defined as ($U, \sigma_x = 4.0$ S/m), geometric ($G, \sigma_{x\,\max} = 10$ S/m) and parabolic ($P, \sigma_{x\,\max} = 10$ S/m) [2]. It can be observed that a reflection coefficient as low as -60 dB is achieved over the entire frequency range of operation. This performance is at least as good as the one reported in [5], which shows variations between -50 to -60 dB over the same frequency range. In fact, comparable performance was expected as 2-D TLM and FDTD have been proved rigorously equivalent [6], [3]. This equivalence for PML, however, has yet to be demonstrated. The advantage of the algorithm presented here is its continuity (non split formulation) as compared to the split formulation used in [5].

IV. CONCLUSION

A new PML–TLM node was presented in the 2-D case (TM mode). Based on a field formulation, which is an extension of the general hybrid node, the new algorithm can be described by a scattering matrix that naturally interfaces with the basic 2-D TLM mesh. In the case where both the computational domain and the PML medium have basic constitutive parameters μ_0 and ε_0, six local incident and reflected voltages are involved. Numerical validations in the case of the modeling of wideband matched load for a WR-28 rectangular waveguide show that excellent performance can be achieved in terms of parasitic reflections at the PML interface.

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